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Power characteristics of planar index-antiguided waveguide lasers with transverse mode competition

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We report comprehensive analysis of output characteristics of homogeneously broadened index-antiguided slab lasers with transverse mode competition. Robust single fundamental mode operation is achieved when the distributive modal loss due to index antiguiding dominates the output coupling loss. Maximal laser efficiency under single fundamental mode operation is investigated numerically for various combinations of single-pass gains and losses. We show analytically that an asymptotic limit of such efficiency exists that is solely determined by the loss ratio between the fundamental and 1st higher-order modes, which equals 66.7% for planar index antiguided lasers. [http://dx.doi.org/10.1063/1.4971307]

It is well known that the output of a laser depends on many parameters in the resonator, such as pumping level and distribution, internal loss, mirror curvatures, output coupling, and cavity length, etc.[1,2] A comprehensive study of laser resonators is therefore essential for understanding these limitations and optimizing their performance. This is especially important for high-power lasers where large mode area (LMA) with robust single fundamental mode is highly desired for high-brightness operation.[3,4] These LMA lasers are mostly multimoded (MM) and require some level of mode discrimination to achieve single fundamental mode operation. Among various means to achieve this goal,[4–9] index antiguiding (IAG) is a relatively simple approach, where the negative index step between the core and the cladding imposes higher loss for higher order modes.[10,11] Robust single fundamental mode oscillation has been reported both in IAG fibers with diameter up to 400 μm[12,13] and IAG planar waveguides with 200-μm core width.[14] Previously we have conducted the first theoretical analysis of output characteristics of fundamentally single-moded (*i.e.*, HOMs can never oscillate) planar IAG lasers with arbitrary laser gain, internal loss, output coupling, and cavity length.[15] However, recent observation of HOM oscillation in a 400-μm-core planar IAG laser[16] indicates the necessity to include transverse mode competition (TMC) due to transverse spatial hole burning[17] in such MM waveguide lasers. Although TMC in MM waveguide lasers has been investigated,[16,18–21] these studies were conducted for specific values or narrow range of gain, loss, and output coupling. A comprehensive study of output characteristics in MM waveguide lasers, not only for IAG but also for LMA waveguide lasers in general, is still lacking. In this work, we conduct comprehensive analysis of output characteristics of single fundamental mode in planar multimoded IAG lasers with arbitrary gain and loss. We report a simple and efficient quasi-analytical method to calculate the threshold gain of the HOM, which is very beneficial to this comprehensive study. We study numerically the extraction efficiency and optimal extraction conditions of the single fundamental mode for various combinations

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of single-pass gains and losses. We present an analytic theory that predicts the absolute maximal extraction efficiency to be 66.7% which is in excellent agreement with numerical modeling.

We consider a generic planar IAG waveguide with a core width d and a length L, sandwiched between two flat mirrors with reflectances R_l and R_r at the left-hand (z = 0) and the right-hand (z = L) ends of the resonator, respectively. The modal loss coefficient α_n of the n^{th} mode is determined by the core width and the refractive indexes of the core and the cladding, and scales as square of the mode order n.[16] For simplicity, we assume the FM and HOM oscillate at the same frequency with a uniform small-signal gain coefficient g_0 in the core region. Previously we have developed a model to calculate the propagating intensities in a homogeneously broadened planar IAG waveguide laser based on a zero-field approximation.[15,22] The forward-propagating average normalized intensities ($I_n^{'+}$) of the fundamental (n=1) and the first higher-order mode (n=2) are governed by the following set of two coupled first-order nonlinear ordinary differential equations[16]

$$\frac{dI_n^{'+}}{dz} = g_0 I_n^{'+} \int_{-d/2}^{d/2} \frac{f_n(x)}{[1 + d\sum_{i=1}^{2} f_i(x)(I_i^{'+} + c_i/I_i^{'+})]} dx - \alpha_n I_n^{'+}, \tag{1}$$

where $c_n = [I_n^{'+}(0)]^2/R_l = R_r[I_n^{'+}(L)]^2$ are mode-specific constants, and $f_1(x) = 2\cos^2(\pi x/d)/d$ and $f_2(x) = 2\sin^2(2\pi x/d)$/d are the normalized intensity profiles across the waveguide core width satisfying $\int_{-d/2}^{d/2} f_n(x)dx = 1$. For arbitrary level of saturation, Eq. (1) needs to be solved self consistently to yield $I_n^{'+}(z)$, from which a multitude of laser output parameters, such as threshold gain, slope efficiency, extraction efficiency, output power, etc., can be derived for individual modes, as was demonstrated in Figs. 1 and 3 of Ref. 16 This process, however, is computationally intensive and is not the easiest way to achieve our goal.

As the focus of the present work is on output characteristics of the fundamental mode, we do not have to calculate $I_2^{'+}$ if we know its threshold gain g_2^{th}. Below we present a simple method to obtain g_2^{th} quasi-analytically. When the unsaturated gain coefficient g_0 equals threshold gain of the fundamental mode g_1^{th}, the FM just starts to oscillate ($I_1^{'+} \approx 0$) while $I_2^{'+}$ is zero. At steady state where the round-trip gain equals the round-trip loss for the FM, Eq. (1) can be integrated to yield the well-known condition for g_1^{th}:

$$g_1^{th} L = \alpha_1 L - \tfrac{1}{2} \ln R_l R_r. \tag{2}$$

At intermediate gain $g_1^{th} < g_0 < g_2^{th}$, the FM oscillates and $I_1^{'+}$ is governed by a single first-order nonlinear ordinary differential equation:

$$\frac{dI_1^{'+}}{dz} = g_0 I_1^{'+} \int_{-d/2}^{d/2} \frac{f_1(x)}{[1 + df_1(x)(I_1^{'+} + c_1/I_1^{'+})]} dx - \alpha_1 I_1^{'+}. \tag{3}$$

This equation is identical to Eq. (9) in Ref. 15 which can be solved much more efficiently than Eq. (1) using the first integral. At the same time, the 1st HOM is below the threshold and integrating Eq. (1) yields

$$g_0 \int_0^L \left(\int_{-d/2}^{d/2} \frac{f_2(x)}{[1 + df_1(x)(I_1^{'+} + I_1^{'-})]} dx \right) dz < \alpha_2 L - \frac{1}{2} \ln R_l R_r. \tag{4}$$

Finally, at $g_0 = g_2^{th}$, the HOM starts to oscillate ($I_2^{'+} \approx 0$) and Eq. (4) becomes

$$g_2^{th} \int_0^L \left(\int_{-d/2}^{d/2} \frac{f_2(x)}{[1 + df_1(x)(I_1^{'+} + I_1^{'-})]} dx \right) dz = \alpha_2 L - \frac{1}{2} \ln R_l R_r. \tag{5}$$

Equation (5) defines g_2^{th}, which can be determined fairly quickly by gradually increasing g_0 in Eq. (3) to obtain $I_1^{'+}$ until a transition from Eq. (4) to Eq. (5) is obtained. We have applied this simplified method to Figs. 1 and 3 of Ref. 16 and obtained excellent agreement.

Equations (2)–(5) are applied to study threshold characteristics of planar IAG waveguide lasers. Consider a common laser configuration where $R_l = 1$ and $R_r = R_{OC}$ (the reflectance of the output coupler). With this notation, output coupling loss is defined as $T = 1 - R_{OC}$, which approximates $-\ln R_{OC}$ when R_{OC} is close to unity. Figure 1(a) shows the contour plot of theoretical threshold gain g_1^{th} of the FM as a function of single-pass loss $\alpha_1 L$ (logarithmic scale) and output coupler reflectance R_{OC} (linear scale). As is apparent from Eq. (2), g_1^{th} decreases monotonically with decreasing distributive loss $\alpha_1 L$ and output coupling loss T (or increasing R_{OC}). Figure 1(b) shows a similar plot for the threshold gain g_2^{th} of the 1st HOM. In the lower-left region where the output coupling loss T dominates the distributive loss $\alpha_1 L$, there is little modal discrimination between the FM and HOM such that HOM oscillates immediately after FM lases. The gain saturation term in Eq. (5) is negligible and g_2^{th} follows the trend of g_1^{th}. This situation is completely different in upper-right region of the figure where $\alpha_1 L >> T$ and gain saturation by the FM suppresses effectively HOM oscillation. We define a robustness parameter $\xi \equiv g_2^{th}/g_1^{th}$ to reflect the robustness of single fundamental mode operation. As shown in Fig. 1(c), to have large ξ and therefore robust single FM operation, one needs to work in the upper-right region where the discriminating IAG loss $\alpha_1 L$ dominates the non-discriminating output coupling loss $T \approx -\ln R_{OC}$ such that gain saturation by the FM effectively suppresses HOM oscillation. It is worth to point out that not all the points along the ξ contour work equally effective in laser optimization. As for the case of plane-wave oscillators, large internal loss significantly reduces lasers' extraction efficiency.[23] This topic is the subject of the following sections.

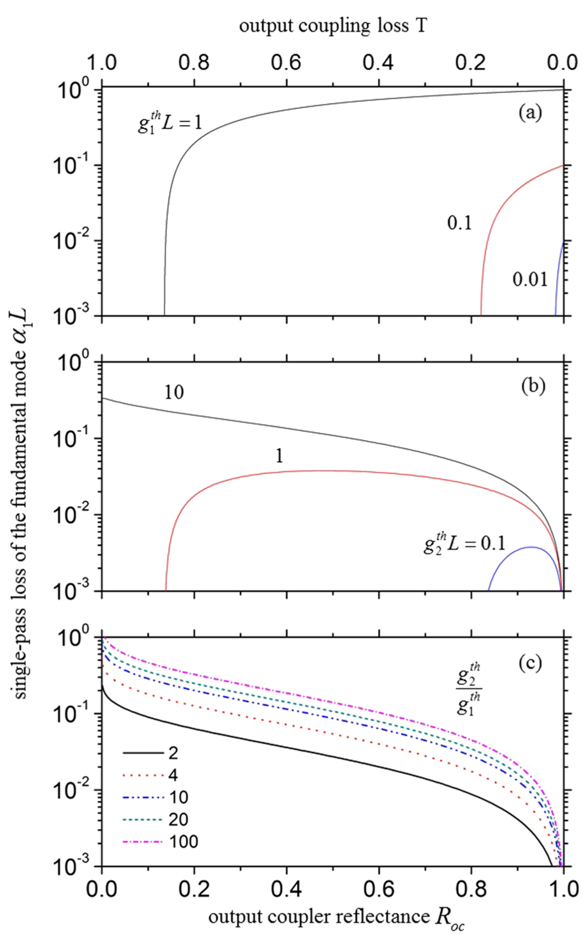

FIG. 1. Contour plots of (a) single-pass threshold gain $g_1^{th}L$ of the fundamental mode, (b) single-pass threshold gain $g_2^{th}L$ of the 1st HOM, and (c) threshold gain contrast g_2^{th}/g_1^{th}, as a function of single-pass loss $\alpha_1 L$ of the fundamental mode (logarithmic scale) and output coupler reflectance R_{OC} (linear scale).

One of the most important performance metrics of lasers is their extraction efficiency. For homogeneously broadened multimoded lasers, the extraction efficiency for the n^{th} mode is defined by[1,2,15]

$$\eta_n = \frac{I_n^{\prime +}(L)(1 - R_{oc})}{g_0 L}. \tag{6}$$

For given gain and loss coefficients, η_n is typically a strong function of the output coupling, and maximal extraction efficiency $\eta_n^{\max}$ exists at some optimal output coupler R_{opt}. For high brightness operation, one is particularly interested in the maximal extraction efficiency $\eta_{SM}^{\max}$ of the single-fundamental mode, which is defined by $\eta_{SM}^{\max}$= maximal η_1 while $\eta_2 = 0$. In principle $\eta_{SM}^{\max}$ can be calculated by solving Eq. (1) to take into account of TMC explicitly, as was done in Ref. 16 With the knowledge of g_2^{th}, however, we propose a simpler method and illustrate its principle below by considering an IAG waveguide with $\alpha_1 L = 0.1$. Firstly we assume a fundamentally single-mode laser by setting $I_2^{\prime +} = 0$ and solve Eq. (3) to obtain η_1 as a function of R_{OC} for selective $g_0 L$, as shown in Fig. 2(a). For each $g_0 L$, $\eta_1^{\max}$ occurs at the critical point where the derivative $\eta_1^{\prime}(R_{opt})$ equals zero and this defines the optimal output coupler R_{opt}. The red dash-dot-dot in Fig. 2(a) denotes the locus of $\eta_1^{\max}$ and its corresponding R_{opt} for different gains. Next, we allow HOM to oscillate (*i.e.*, $I_2^{\prime +}$ can be non-zero) and identify single FM region in Fig. 2(a). To do so, we calculate g_1^{th} and g_2^{th} as a function of R_{OC} for $\alpha_1 L$ = 0.1, which is displayed in Fig. 2(b). The regions below the g_1^{th} curve, between g_1^{th} and g_2^{th} curves, and above the g_2^{th} curve, represent no oscillation, FM only, and multimode oscillation, respectively. While the g_1^{th} curve is monotonic, the g_2^{th} curve has a local minimum at $g_0 L$ = 2.654, below which the oscillation is single FM for all R_{OC}. For each $g_0 L$ > 2.654, the g_2^{th} curve defines two R_{OC} values separating the single FM from MM operation. The g_2^{th} curve in Fig. 2(b) can then be mapped into Fig. 2(a) as the blue dash-dot curve, which also represents η_1 at the threshold of the 1st HOM. Finally, we can define $\eta_{SM}^{\max}$ as follows, which is represented by black circles in Fig. 2(a): Below the g_2^{th} curve the laser is single fundamental mode so $\eta_{SM}^{\max} = \eta_1^{\max}$; above it the laser is MM and $\eta_{SM}^{\max}$ follows the g_2^{th} curve, as HOM kicks in before the laser reaches to $\eta_1^{\max}$. These two segments intersect at $g_0 L$ = 4.235 which defines a sharp kink. Figure 2(a) clearly shows that, as a result of TMC, $\eta_{SM}^{\max}$ is suppressed in regions of high gain and the corresponding optimal output coupler R_{opt} increases.

Figure 2(a) can be repeated to obtain $\eta_{SM}^{\max}$ and R_{opt} for different values of $\alpha_1 L$. The result is summarized in Fig. 3(a). For comparison, the same calculation for a fundamentally single-mode IAG laser is displayed in Fig. 3(b).[15] As shown, without the HOM competing with the FM, all curves of constant $g_0 L$ (solid line) or $\alpha_1 L$ (dash line) are smooth and $\eta_1^{\max}$ approaches unity in the limit of low loss. With the HOM competing with the FM, the trend remains the same in high-loss regions while it is squeezed downwards in low-loss regions. The squeezing results in kinks in all curves of

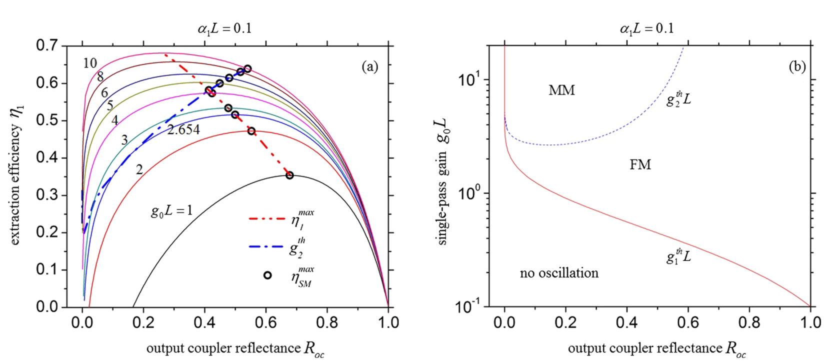

FIG. 2. For $\alpha_1 L = 0.1$, (a) extraction efficiency of the fundamentally single mode vs. output coupling for selected fixed gains, and (b) threshold gain vs. output coupling for the FM and 1st HOM in a MM IAG laser. In (a), red dash-dot-dot is the locus of maximal extraction efficiency of the FM, blue dash-dot is η_1 at the threshold of 1st HOM, and black circle is the maximal single-fundamental-mode extraction efficiency in a MM IAG laser.

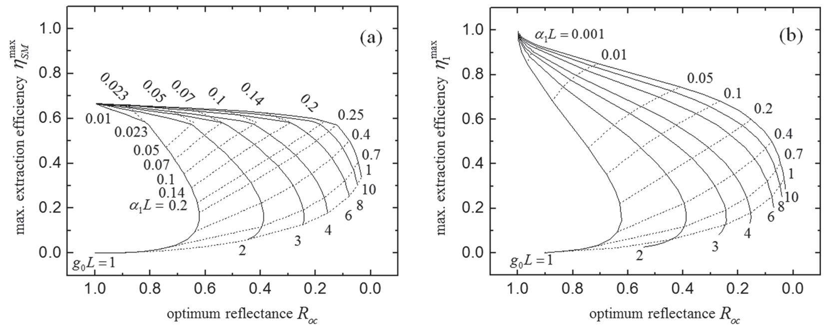

FIG. 3. (a) Maximal extraction efficiency of the single fundamental mode in a multimoded laser ($\eta_{SM}^{\max}$) and optimal output coupler reflectance R_{opt}, for various single-pass gains (solid lines) and distributive losses (dash lines). (b) Same as (a) except maximal extraction efficiency of a fundamentally single-moded laser ($\eta_1^{\max}$).

constant gain or loss (notice that kinks in curves of very low gain or very high loss are not shown). Specifically, the dash curve with $\alpha_1 L = 0.1$ in Fig. 3(a) is identical to the $\eta_{SM}^{\max}$ curve in Fig. 2(a). Figure 3 clearly indicates that $\eta_{SM}^{\max}$ is suppressed and approaches an asymptotic value of 67%, which defines an absolute maximal extraction efficiency $H_{SM}^{\max}$ of the laser under single fundamental mode operation (H stands for capital Greek letter η). The squeezing also makes $\eta_{SM}^{\max}$ insensitive to (internal) distributed loss and (external) output coupling loss at high gain – a property that is also shared by plane-wave resonators.[2]

The value of $H_{SM}^{\max}$ can be derived analytically as follows. Referring to Fig. 3(a), $H_{SM}^{\max}$ equals $\eta_{SM}^{\max}$ in the limit of low distributive loss, weak output coupling, and high gain. This point corresponds to the largest $\eta_{SM}^{\max}$ in Fig. 2(a), which occurs at the intersection of the g_2^{th} curve and the η_1 curve of the highest $g_0 L$. It therefore satisfies both Eq. (5) and

$$g_2^{th}\int_0^L\left(\int_{-d/2}^{d/2}\frac{f_1(x)}{[1+df_1(x)(I_1'^{+}+I_1'^{-})]}dx\right)dz=\alpha_1 L-\frac{1}{2}\ln R_{OC}. \tag{7}$$

Equation (7) states simply that round-trip gain equal to round-trip loss for the FM at g_2^{th}. Let $\alpha_2 = k\alpha_1$ where $k = 4$ for planar IAG waveguides. Multiplying Eq. (7) by k and subtracting Eq. (5) yields

$$g_2^{th}\int_0^L\int_{-d/2}^{d/2}\frac{kf_1(x)-f_2(x)}{[1+df_1(x)(I_1'^{+}+I_1'^{-})]}dxdz=-\frac{(k-1)}{2}\ln R_{OC}. \tag{8}$$

At the limit of low loss $\alpha_1 L \to 0$ and weak coupling $R_{OC} \to 1$ where $I_1'^{+} \approx I_1'^{-}$ and $-\ln R_{OC} \approx 1 - R_{OC}$, Eq. (8) is reduced to

$$g_2^{th}L\int_{-d/2}^{d/2}\frac{kf_1(x)-f_2(x)}{[1+df_1(x)2I_1'^{+}]}dx\approx\frac{(k-1)}{2}(1-R_r). \tag{9}$$

Since the right-hand side of Eq. (9) is close to 0, we have $df_1(x)2I_1'^{+} >> 1$ and Eq. (9) becomes

$$\frac{g_2^{th}L}{2I_1'^{+}}(k-2)=\frac{(k-1)}{2}(1-R_r). \tag{10}$$

The maximum extraction efficiency $H_{SM}^{\max}$ is therefore

$$H_{SM}^{\max}=\frac{I_1'^{+}(1-R_r)}{g_2^{th}L}=\frac{k-2}{k-1}. \tag{11}$$

With $k = 4$ for planar IAG lasers, $H_{SM}^{\max} = 2/3 \sim 0.67$, which agrees well with the numerical result shown in Fig. 3(a). Experiments are currently underway to confirm this prediction.

In conclusion, we have conducted comprehensive analysis of output characteristics of homogeneously broadened IAG planar waveguide lasers with arbitrary single-pass gain and loss. Specifically, our modeling takes into account transverse mode competition due to transverse spatial hole burning. We propose an efficient semi-analytical method to calculate threshold gains of higher-order modes. We show that robust single fundamental mode operation can be obtained in IAG lasers when distributive modal loss dominates output coupling loss. We study laser extraction efficiency and optimal output coupling for various combinations of single-pass gains and losses. Drastically different from a fundamentally single-mode laser where the maximal extraction efficiency can approach unity, the extraction efficiency of the single fundamental mode in a MM laser approaches an asymptotic value. We present an analytic theory to show that this limiting value is solely determined by the modal loss ratio between the FM and the HOM. Our theory predicts a value of 2/3 for planar IAG waveguides, which agrees very well with numerical modeling. Our results have important implications to the design and optimization of IAG waveguide lasers. Our methods can be readily extended to other LMA lasers with different modal loss discrimination mechanisms, as well as to the study of general aspects of mode competition in multimoded systems.

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